International Journal of Engineering, Science and Mathematics (UGC Approved)
Vol. 6 Issue 8, December 2017,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gate as well as in Cabell's Directories of Publishing Opportunities, U.S.A

ANALYSIS OF SUPPLY CHAIN INVENTORY MODEL WITH SELLING PRICE DEPENDENT DEMAND RATE

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ABSTRACT

Most of the researchers in inventory system were directed towards non-deteriorating products. However, there are certain substances, whose utility do not remain same with the passage of time. Deterioration of these items plays an important role and items cannot be stored for a long time. Deterioration of an item may be defined as decay, evaporation, obsolescence, loss of utility or marginal value of an item that results in the decreasing usefulness of an inventory from the original condition. When the items of the commodity are kept in stock as an inventory for fulfilling the future demand, there may be the deterioration of items in the inventory system, which may occur due to one or many factors i.e. storage conditions, weather conditions or due to humidity. Many businesses are not as successful as they could be simply because they lack the know-how or the will to implement sound inventory management and control practices. Successful inventory is a compromise between low inventory levels and meeting targeted fill rates. Investing in the right inventory and reducing excess will improve customer fill rates, inventory turnover and cash flow and profits.

Keywords: Deterioration, non-deteriorating products, inventory

INTRODUCTION

Supply chain management involves the entire process of planning, implementing and controlling supply chain operations. It is not just the process of order goods and receiving them into inventory, but making certain that they are shipped and delivered to customers in a timely fashion. That means that those in the procurement areas of each company are responsible for all aspects of goods movement beginning with the purchase requisition and ending with the delivery of finished goods to the customer. In the case of a manufacturing company, this process will also involve the procurement of the raw goods and work-in-process phase of the manufacturing process. Supply chain has become a vital topic in management science and industry. The logical progression of the inventory model is to investigate the supply chain that consists of suppliers, manufacturers, distributors and

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retailers. Each one of them holds inventory in some form to support the requirement of the customer at the end of the chain. In supply chain many problems still need a careful consideration regarding solution procedure to support respectivesystems.

REVIEW OF LITERATURE

Weng and Wong (1993) developed a general all-unit quantity discount model for determining the optimal pricing and replenishment policy under the condition of price sensitive demand. Weng (1995) later considered the vendor's quantity discount from the perspective of reducing the vendor's operating cost and increasing the buyer's demand. Van der Heijden et al. (1997) presented stock allocation policies in general single-item and N- echelon distribution systems, where it is allowed to hold stock at all levels in the network. Ha and Kim (1997) used a graphical method to analyze the integrated vendor-buyer inventory status to derive an optimal solution. Hwang and Shinn (1997) studied effects of permissible delay in payments on retailer's pricing and lot sizing policy for exponentially deteriorating products. Diks and De Kok (1998) determined a cost optimal replenishment policy for a divergent multi-echelon inventory system. A joint replenishment policy for multi- echelon inventory control was proposed by Axsater and Zhang (1999). Wang et al. (2000) analyzed supply chain models for perishable products under inflation and permissible delay in payment. A multi-echelon inventory model for a deteriorating item was developed by Rau et al. (2003). An optimal joint total cost has been derived from an integrated perspective among the supplier, the producer and the buyer. Adeteriorating item inventory model in a supply chain was proposed by Wu and Wee (2005). Shortages in inventory were allowed and fully backlogged. Two-echelon inventory model with lost sales was proposed by Hill et al. (2007). Singh (2008) have recently coordinated perishable inventory model with quadratic demand. **Zhou** (2009) considered a note on an EOQ model with stock and price sensitive demand. **None of them consider the replenishment** policy with finite replenishment rate and price sensitive demand for deteriorating item.

ASSUMPTIONS AND NOTATIONS

The mathematical model in this study is developed on the basis of the following assumptions:

- There are single vendor and a single buyer for a single product in thismodel.
- The production rate is finite and is greater than the sum of all the buyerdemand.
- There is no replacement or repair of deterioratedunits.
- The cost of a backorder includes a fixed cost and a cost is proportional to the length of time for which backorder exist.
- The cost of a lost sale, excluding the lost of profit, is constant (goodwillcost).
- Shortages are allowed for buyer only, which is partially backlogged.

The following notation is assumed:

θ The constant deterioration rate($0<t<1$)

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D	Demand rate for vendor, $D = \alpha P^{\beta}$ where c, d are positive constants
R	Demand rate for buyer, $R = \alpha P^{\beta^{v}}$ where c, d are positive constants
KD	The production rate per year, whereK>1
T	Time length of each cycle, where $T = T_1 + T_2$
T_1	The length of production time in each productioncycle
T_2	The length of non production time in each production cycle.
$I_{v1}(t_1)$	Inventory level for vendor when t_1 is between 0 and T_1
$I_{v2}(t_2)$	Inventory level for vendor when t_2 is between 0 andT_2
$I_b(t)$	Inventory level for buyer when t is between 0 and t_1
$I_{b1}(t)$	Inventory level for buyer when t is between t_1 and T/ n
I_{mv}	The maximum inventory level forvendor
I_{mi}	The maximum inventory level forbuyer
S_v	The setup cost for each production cycle forvendor
S_b	The setup cost per order for buyer
$(F_v + \phi t)$	Holding cost per unit time for vendor
$(F_b + \phi t)$	Holding cost per unit time forbuyer
C_v	Deterioration cost per unit time for vendor
C_b	Deterioration cost per unit time for buyer
K_b	Shortage cost per unit time forbuyer
O_b	Opportunity cost per unit time for buyer
P_v	Vendor's retailprice
p_v	The unit production cost forvendor
p_b	The unit price for buyer
w_0	Fixed shortage cost per/ independent time(≥ 0)
w	Fixed shortage cost per unitbackordered

π_0Goodwill cost of a lost sale, that is, the cost derived of a lost sale excluding the lost of profit(≥ 0)

ψ	Length of the inventory cycle over which the net stock is less that or equal to zero, that is, length of shortage cycle(≥0)
VC	The cost of vendor per unittime
BC	The cost of buyer per unittime
TC	The integrated cost of vendor and all buyer per unit time

MATHEMATICAL ANALYSIS

The Model for Single vendor and Single buyer

The vendor inventorymodel

The cycle time interval is T, it can be divided into two periods: the production period during T_1 and the non- production period duringT_2.

The inventory system is represented by the following differential equations:

$$I'_{v1}(t_1)+\theta I_{v1}(t_1)=\alpha P_v^{\beta}(K-1), \quad 0\le t_1\le T_1 \qquad \text{....(1.1)}$$

$$I'_{v2}(t_2)+\theta I_{v2}(t_2)=-\alpha P_v^{\beta}, \quad 0\le t_2\le T_2 \qquad \text{....(1.2)}$$

Variousboundaryconditions$I_{v1}(0)=0, I_{v2}(T_2)=0$,thesolutionsoftheabovedifferentialequationsare

$$I_{v1}(t_1)=\frac{(K-1)\alpha P_v^{\beta}}{\theta}\left(1-e^{-\theta t_1}\right), \quad 0\le t_1\le T_1 \qquad \text{....(1.3)}$$

$$I_{v2}(t_2)=\frac{\alpha P_v^{\beta}}{\theta}\left(\frac{e^{\theta T_2}-e^{\theta t_2}}{e^{\theta t_2}}\right), \quad 0\le t_2\le T_2 \qquad \text{....(1.4)}$$

From (1.4)

$$I_{mv}=\frac{\alpha P_v^{\beta}}{\theta}\left(e^{\theta T_2}-1\right) \qquad \text{....(1.5)}$$

when $I_{mv} = I_{v2}(0)$

By the boundary condition, $I_{v1}(T_1)= I_{v2}(0)$, one can drive the following equation:

$$T_1=\frac{1}{K-1}T_2 \qquad \text{....(1.6)}$$

Knowing

$T=T_1+T_2$ ….(1.7)

one can derive

$$T=\frac{K}{K-1}T_2 \quad \text{….(1.8)}$$

The buyer inventory model

In this article, the inventory system goes like this: I_{mi} units of item arrive at the inventory system at the beginning of each cycle. During the time interval [0, t_1], the inventory level is dropping to zero due to demand and deterioration. Then shortage interval keeps to the end of the current ordercycle.

During the time interval [0, t_1], the interval level decreases owing to price sensitive demand rate as well as deterioration. Thus, the differential equation representing the inventory status is given by

$$I_b'(t)+\theta I_b(t)=-\alpha P_b^{\beta}, \quad 0\le t\le t_1 \quad \text{….(1.9)}$$

withtheboundaryconditions$I_b(t_1)=0$.ThesolutionofEq.(1.9)is

$$I_b(t)=\frac{\alpha P_b^{\beta}}{\theta}\left(e^{\theta(t_1-t)}-1\right), \quad 0\le t\le t_1 \quad \text{….(1.10)}$$

From (1.10)

$$I_{mi}=\frac{\alpha P_b^{\beta}}{\theta}\left(e^{\theta t_1}-1\right) \quad \text{….(1.11)}$$

During the shortage interval $\left[t_1, \frac{T}{n}\right]$, the demand at time t is partially backlogged at fraction $B\left(\frac{T}{n}-t\right)$. Thus,

the inventory level at time t, is governed by the following differential equation:

$$I_{b1}'(t)=\frac{-\alpha P_b^{\beta}}{1+\delta\left(\frac{T}{n}-t\right)} \quad t_1\le t\le \frac{T}{n} \quad \text{….(1.12)}$$

withtheboundaryconditions$I_{b1}(t_1)=0$.ThesolutionofEq.(1.12)is

$$I_{b1}(t)=\frac{-\alpha P_b^{\beta}}{\delta}\left\{\ln\left[1+\delta\left(\frac{T}{n}-t\right)\right]-\ln\left[1+\delta\left(\frac{T}{n}-t\right)\right]\right\}, \quad t_1\le t\le \frac{T}{n} \quad \text{….(1.13)}$$

$b1 \quad \lfloor \lfloor \quad n \quad 1 \rfloor \quad \lfloor \quad n \quad \rfloor\rfloor \quad 1 \quad n$

Putting $t = T/n$ in Eq. (1.16), we obtain the maximum amount of demand backlogged per cycle as follows:

$$= - \; T \; \alpha P^{\beta} \left[\quad T \quad \right] \qquad \text{….(1.14)}$$

$$S \quad I_{b1}\left(\frac{}{n}\right) = \frac{\Box_b}{\delta} \ln \left| 1 + \delta\left(\frac{}{n} - t_1 \right) \right|$$

From Eq. (1.11) and (1.14), we can obtain the order quantity, Q, as

$Q = I_{mi} + S$

$$= \frac{\alpha P^{\beta}_b}{\theta}\left(e^{\theta t_1} - 1\right) + \frac{a}{\delta} \ln \left[1 + \delta\left(\frac{T}{n} - t_1 \right) \right] \qquad \text{….(1.15)}$$

Next, the relevant inventory cost per cycle for vendor consists of the following three elements:

The setup cost per cycle is

S_v. ….(1.16)

The inventory holding cost per cycle is

$$HC_v = p_v \left[\int_0^{T_1} (F_v + \phi t) I_{v1}(t_1) dt_1 + \int_0^{T_2} (F_v + \phi t) I_{v2}(t_2) dt_2 - \int_0^{t_1} (F_v + \phi t) I_b(t) dt \right] \qquad \text{….(1.17)}$$

The deterioration cost per cycle is

$$DC_v = \theta C_v \left[\int_0^{T_1} I_{v1}(t_1) dt_1 + \int_0^{T_2} I_{v2}(t_2) dt_2 - \int_0^{t_1} I_b(t) dt \right] \qquad \text{….(1.18)}$$

The purchase cost per cycle is

$$PC_v = p_v K \alpha P^{\beta} T_{v1} \qquad \text{….(1.19)}$$

The vendor's total cost is the sum of (1.16), (1.17), (1.18), (1.19) and (1.26) as

VC = holding cost + deterioration cost + ordering cost + purchase cost -

– buyer'spurchasecost ….(1.20)

Next, the relevant inventory cost per cycle for buyer consists of the following five elements:

The setup cost per cycle is

nS_b. ….(1.21)

The inventory holding cost per cycle is

$$HC_b = p_b \int_0^{t_1} (F_b + \varphi t) I_b(t)\,dt \qquad \text{….(1.22)}$$

The deterioration cost per cycle is

$$DC_b = \theta C_b \int_0^{t_1} I_b(t)\,dt \qquad \text{….(1.23)}$$

The shortage cost per cycle is

$$SC_b = K_b \int^{T/n} [-I_{b1}(t)]dt$$

The opportunity cost per cycle is

$$OC_b = O_b \int_{t_1}^{T/n} \alpha P_b^{\beta}[1-B_b]dt \qquad \text{....(1.25)}$$

The purchase cost per cycle is

$$PC_b = p_b Q \qquad \text{....(1.26)}$$

The buyer's total cost is the sum of (1.21), (1.22), (1.23), (1.24), (1.25) and (1.26) as

BC = holding cost + deterioration cost + shortage cost + opportunity cost + ordering cost +purchase cost – customer'spurchasecost ...(1.27)

The integrated joint total cost function TC for the vendor and the buyer is the sum of VC and BC. From (1.20) and (1.27),

The integrated total cost can be written as

TC=VC+BC ....(1.28)

As can be plainly observed this is a function of a continuous variable T_2, t_1.

CONCLUSION

Integrated inventory model for decaying items with price sensitive demand over an infinite-horizon has been developed. There is no doubt that price affect demand. In the case of price elasticity of demand it is used to see how sensitive the demand for a good is to a price change. The higher the price elasticity, the more sensitive consumers are to price changes. Very high price elasticity suggests that when the price of a good goes up, consumers will buy a great deal less of it and when the price of that good goes down, consumers will buy a great deal more. The fraction of backlogged demand is described by a function which depends on the waiting time before receiving the item and on the length of the inventory cycle over which the net stock is not positive. The model has been solved numerically. It is observed that as the difference between vendor's retail price and buyer's retail price increases, total cost increases. As should have been, it proves that less difference in their retail prices to be financially better. The reason being very obvious, that in today's time with so much volatility around, a model with same retail price for both vendor and buyer just does not sound feasible.

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